Noninstantaneous collisions and two concepts of quasiparticles

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The kinetic theory recently implemented in heavy ion reactions combines a nonlocal and noninstantaneous picture of binary collisions with quasiparticle features. We show that the noninstantaneous description is compatible with the spectral concept of quasiparticles while the commonly used variational concept is consistent only with instantaneous collisions. The rearrangement energy, by which the variational concept surpasses the spectral one, is shown to be covered by a medium effect on noninstantaneous collisions.
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The quasiparticle concept provides a basic theoretical framework for description of interacting Fermi systems. Although quasiparticles are well defined only close to the ground state, i.e., at small temperatures and under weak perturbing fields, a lack of tractable theories for systems far from equilibrium forces physicists to deal with quasiparticles also in this region. From a number of highly nonequilibrium systems treated in the quasiparticle picture one should, perhaps, mention heavy ion reactions where the excitation energy far exceeds the Fermi energy.

It is disturbing that an extension of the quasiparticle concept far from the ground state is not unique. There are two formulations of the quasiparticle concept, the phenomenologic and the microscopic. The former relies on the variation of thermodynamic quantities, the latter on properties of the single-particle spectrum. Close to the ground state these concepts are proven to be identical but they become increasingly distinct as an excitation of the system increases. A natural question arises: Which concept of quasiparticles works better in kinetic equations for highly nonequilibrium systems? As we show in this Rapid Communication, the answer depends on approximations employed in the collision integral. If the collisions are treated as local and instantaneous, as is common in equations of Boltzmann type, the variational concept seems to be better, at least with respect to processes of a thermodynamic and hydrodynamic character. In contrast, a noninstantaneous treatment of collisions, as suggested by Danielewicz and Pratt [1], is compatible only with the spectral concept.

Let us rehash first both concepts. In the original phenomenologic formulation, Landau [2,3] postulates that a density of quasiparticles equals the density of composing particles,

$$n=\int \frac{dk}{(2\pi)^3}\tilde{f}_k, \tag{1}$$

where $\tilde{f}_k$ is the quasiparticle distribution in momentum space. Assuming furthermore that quasiparticles cover all degrees of freedom of the system, the quasiparticle energy $\tilde{\epsilon}$ can be defined as a variation of the total energy $\mathcal{E}$,

$$\tilde{\epsilon}_k=\frac{\delta\mathcal{E}}{\delta\tilde{f}_k}. \tag{2}$$

From the entropy follows that the equilibrium distribution is of the Fermi-Dirac form, $\tilde{f}_k=f_{FD}(\tilde{\epsilon}_k-\tilde{\mu})$. Out of equilibrium, $\tilde{f}_k$ differs from the Fermi-Dirac distribution and relations (1) and (2) hold locally in time and space.

The microscopic Green's function foundations [3–5] of the quasiparticle theory define the quasiparticle energy alternatively from a singularity of the single-particle spectral function

$$A=\frac{\Gamma}{\left(\omega-\frac{k^2}{2m}-\Sigma\right)^2+\frac{1}{4}\Gamma^2}, \tag{3}$$

where Σ and $\frac{1}{2}\Gamma$ are real and imaginary parts of the self-energy. For long-lived excitations, $\Gamma\to 0$, the spectral definition of the quasiparticle energy reads

$$\epsilon_k=\frac{k^2}{2m}+\Sigma(\epsilon_k,k). \tag{4}$$

The quasiparticle distribution is identified from a singularity of the single-particle correlation function $G^<$. In equilibrium, where $G^<(\omega,k)=f_{FD}(\omega)A(\omega,k)$, this approach also yields the Fermi-Dirac distribution $f_k=f_{FD}(\epsilon_k-\mu)$. Again, out of equilibrium all relations are locally valid, except for the Fermi-Dirac form of f_k.

Various studies [3,6,7] prove that close to the ground state Landau's quasiparticles are the elementary excitations seen in the single-particle spectrum. In these proofs it is essential that dissipative processes freeze out with the square of the temperature so that quasiparticles become free of collisions. Far from the ground state, however, the collisions become important. As long as the collisions are local and instantaneous, as is the case for a weak interaction, they have no effect on the thermodynamic properties and both quasiparticle concepts remain equivalent. When nonlocal, the collisions affect thermodynamic properties in a way that escapes

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the variational approach. A clear-cut example is the system of hard spheres whose density and total energy do not differ from the ideal gas and yet its equation of state includes the virial corrections known as the inaccessible volume. The noninstantaneous collisions result in even deeper changes on which we focus in this paper.

We will discuss two groups of virial corrections due to which the spectral quasiparticles do not satisfy postulates of the variational concept. Both are caused by the finite duration of collisions. First, the density of spectral quasiparticles differs from the density of composing particles by the correlated density, $n^{\rm cor}=n-\int[dk/(2\pi)^3]f_k$, as is known from the Bethe-Uhlenbeck virial expansion [8,9]. Second, the variational quasiparticle energy $\tilde{\epsilon}$ differs from the spectral one by the rearrangement energy $\epsilon^{\rm re}=\tilde{\epsilon}-\epsilon$. This has been observed earlier [10] for the Galitskii-Feynman approximation used widespread, either to evaluate the total energy $\mathcal{E}$ as the starting point of the variational approach or to evaluate the self-energy needed in the spectral approach. It should be noted that the Galitskii-Feynman approximation is missing the particle-hole channels important at very low temperatures. Our discussion is thus limited to rather highly excited systems.

Collisions of quasiparticles have been recently studied in [11] with the help of methods developed for gases [12,13]. A kinetic equation derived from nonequilibrium Green's functions by a systematic gradient expansion includes a scattering integral in which collisions are described as nonlocal and noninstantaneous events. Although small in slowly varying systems, these nonlocal and noninstantaneous corrections appreciably influence a behavior of the system, since they affect conserving quantities and therefore contribute to thermodynamic and hydrodynamic properties. For instance, the nonlocality corrects for the inaccessible volume and the finite duration yields the correlated density. On top of these two effects known from the theory of gases, the found collisions possess another feature in that the momentum and the energy of a colliding pair of quasiparticles do not conserve. Small amounts of momentum and energy are gained by a colliding pair due to changes of the Pauli blocking during the collision.

The transfer of momentum and energy between the colliding pair and the medium of background particles provides an important link between the variational and the spectral concepts of quasiparticle energies. Since we want to focus on this energy balance, we assume a homogeneous system for simplicity. We will show that the rearrangement energy contributing to the variational energy simulates for the energy gained by a pair of quasiparticles during in-medium collisions.

The variational approach works only if binary collisions are treated within the instantaneous and elastic approximation. To show why, assume a phenomenological kinetic equation

$$\frac{\partial\tilde{f}_k}{\partial t}=\tilde{I}_k, \tag{5}$$

where $\tilde{I}_k$ is an unspecified collision integral. To conserve the number of particles,

$$\frac{dn}{dt}=\int\frac{dk}{(2\pi)^3}\frac{\partial\tilde{f}_k}{\partial t}=\int\frac{dk}{(2\pi)^3}\tilde{I}_k, \tag{6}$$

the collision integral has to satisfy $\int dk\tilde{I}_k=0$, which is possible only for instantaneous collisions. In addition, from the energy balance,

$$\frac{d\mathcal{E}}{dt}=\int\frac{dk}{(2\pi)^3}\frac{\delta\mathcal{E}}{\delta\tilde{f}_k}\frac{\partial\tilde{f}_k}{\partial t}=\int\frac{dk}{(2\pi)^3}\tilde{\epsilon}_k\tilde{I}_k, \tag{7}$$

it follows that the total energy conserves, $d\mathcal{E}/dt=0$, only if the sum of variational quasiparticle energies conserve in collisions, $\int dk\tilde{\epsilon}_k\tilde{I}_k=0$.

In contrast, the kinetic equation resulting from the spectral concept as an asymptotic of nonequilibrium Green's function in the Galitskii-Feynman approximation [11],

$$\begin{aligned}\frac{\partial f_k}{\partial t}=&\int\frac{dpdq}{(2\pi)^5}\delta(\epsilon_k+\epsilon_p-\epsilon^-_{k-q}-\epsilon^-_{p+q}-2\Delta_E)\\&\times|T^-|^2(1-f_k-f_p)f^-_{k-q}f^-_{p+q}\\&-\int\frac{dpdq}{(2\pi)^5}\delta(\epsilon_k+\epsilon_p-\epsilon^+_{k-q}-\epsilon^+_{p+q}+2\Delta_E)\\&\times|T^+|^2f_kf_p(1-f^+_{k-q}-f^+_{p+q}),\end{aligned} \tag{8}$$

has a noninstantaneous collision integral that does not conserve the sum of quasiparticle energies. For distributions we use abbreviations $f=f(t)$ and $f^\pm=f(t\pm\Delta_t)$, and similarly for arguments of quasiparticle energies. The T matrix, $T_R=|T|e^{i\phi}$, is centered between asymptotic states, $T^\pm=T^\pm(\epsilon_1^\pm+\epsilon_2^\pm\pm\Delta_E,k,p,q,t\pm\frac{1}{2}\Delta_t)$. Apparently, the collision delay, $\Delta_t=\partial\phi/\partial\Omega|_{\Omega=\epsilon_1+\epsilon_2}$, and the energy gain of a colliding pair, $2\Delta_E=-\partial\phi/\partial t|_{\Omega=\epsilon_1+\epsilon_2}$, do not meet phenomenologic expectations about the structure of the collision integral.

The difference between Eqs. (5) and (8) is even more obvious from the conservation laws found in [11] from Eq. (8) by integration over momentum k with factors 1 and ϵ_k. The balance of the number of particles,

$$\frac{dn}{dt}=\frac{d}{dt}\int\frac{dk}{(2\pi)^3}f_k+\frac{d}{dt}\int dP\Delta_t, \tag{9}$$

where

$$\begin{aligned}dP=&\frac{dkdpdq}{(2\pi)^8}|T|^2\delta(\epsilon_k+\epsilon_p-\epsilon_{k-q}-\epsilon_{p+q})\\&\times f_kf_p(1-f_{k-q}-f_{p+q}),\end{aligned} \tag{10}$$

includes the term proportional to the collision delay. This is exactly the correlated density, $n^{\rm cor}=\int dP\Delta_t$, found in [9] from the equilibrium Green's functions. The kinetic equation (8) thus implies that the number of spectral quasiparticles does not equal the number of composing particles.

The energy balance found from Eq. (8),

$$\frac{d\mathcal{E}}{dt}=\int\frac{dk}{(2\pi)^3}\epsilon_k\frac{\partial f_k}{\partial t}+\frac{d}{dt}\int dP\,\frac{\epsilon_k+\epsilon_p}{2}\Delta_t-\int dP\Delta_E, \tag{11}$$

also includes contributions that are not compatible with the phenomenologic postulates. Similar to the correlated density, there is the energy of correlated particles $\propto\Delta_t$. Moreover, due to the transfer between the background and the colliding pair, there is a mean energy gain $\propto\Delta_E$ by which the energy covered by single-particle degrees of freedom (the sum of quasiparticle energies) can change in time.

One might wonder whether the kinetic equation (8) conserves the energy at all because the right-hand side of balance equation (11) does not have a transparent form of the total time derivative. Although it is a tedious task, it can be shown that Eq. (8) conserves the energy given by

$$\mathcal{E}=\int\frac{dk}{(2\pi)^3}f_k\frac{k^2}{2m}+\frac{1}{2}\int\frac{dkdp}{(2\pi)^6}f_kf_p\,\mathrm{Re}\,T_R(\epsilon_k+\epsilon_p,k,p,0)+\int dP\,\frac{\epsilon_k+\epsilon_p}{2}\Delta_t. \tag{12}$$

One can check that Eq. (11) results from the time derivative of Eq. (12). The energy of correlated particles directly corresponds to the second term of Eq. (11). The second term of Eq. (12) splits into the self-energy part of the quasiparticle energy and into the mean energy gain. The mean energy gain follows exclusively from the time derivative of $\mathrm{Re}\,T_R$ in agreement with fact that the effect of medium on the binary collision is responsible for the energy transfer between colliding particles and the background. We note that the total energy (12) is identical with the Galitskii-Feynman approximation in the limit of small scattering rates. Although formula (12) holds out of equilibrium, we want to outline its simple derivation for the equilibrium case. In the general expression for the energy [14],

$$\mathcal{E}=\int\frac{dkd\omega}{(2\pi)^4}\frac{1}{2}\left(\omega+\frac{k^2}{2m}\right)f_{FD}(\omega)A(\omega,k),$$

one employs the limit of small scattering rates [7,9,11,13,15],

$$A=\left(1+\frac{\partial\Sigma}{\partial\omega}\right)2\pi\delta(\omega-\epsilon_k)+\mathrm{Re}\,\frac{\Gamma}{(\omega-\epsilon_k+i0)^2}.$$

The second term represents off-shell contributions neglected within the so-called quasiparticle approximation commonly used to derive the quasiparticle theory from Green's functions. Its inclusion is essential for all correlated quantities.

Being able to cover the correlated density and the latent heat due to the mean energy gain, the spectral concept provides a more sophisticated description of interacting Fermi liquids than the phenomenological one. This superiority, however, is at the cost of such complex corrections as the collision delay and energy gain during collisions. It is interesting to see under what conditions the theory based on the spectral concept reduces to the phenomenologic one.

According to Eq. (6), the phenomenologic concept works only if the collision duration is negligible. Sending $\Delta_t\to0$ in Eq. (8) one obtains the instant collision integral and consequently no correlated density appears in the number of particles balance, $\int dP\Delta_t\to0$. At least with respect to the density of quasiparticles one can say that the distribution of quasiparticles becomes close to the variational distribution,

$$f_k\to\tilde{f}_k. \tag{13}$$

In the energy conservation (11) and the total energy (12), the neglect of the collision delay suppresses the contribution of the correlated particles, $\int dP(\epsilon_k+\epsilon_p)\Delta_t\to0$. The total energy which conserves within the instantaneous but still nonelastic approximation of collisions, has the familiar form [16]

$$\mathcal{E}\to\int\frac{dk}{(2\pi)^3}\tilde{f}_k\frac{k^2}{2m}+\frac{1}{2}\int\frac{dkdp}{(2\pi)^6}\tilde{f}_k\tilde{f}_p\times\mathrm{Re}\,T_R(\epsilon_k+\epsilon_p,k,p,0), \tag{14}$$

which is commonly used as a starting point in variational approaches [10].

In spite of the above similarities, it would be premature to conclude that in the limit of instantaneous collisions, $\Delta_t\to0$, the spectral and the variational approaches are identical. The energy balance (11) in the instantaneous approximation

$$\frac{d\mathcal{E}}{dt}\to\int\frac{dk}{(2\pi)^3}\epsilon_k\frac{\partial\tilde{f}_k}{\partial t}-\int dP\Delta_E, \tag{15}$$

still includes the mean energy gain $\int dP\Delta_E$. There is a simple but incorrect argument that the mean energy gain can be neglected as $\Delta_t\to0$. (The energy gain follows from the time dependency of scattering phase shift, i.e., at the end, from the time dependency of a distribution of particles in the background. In the instantaneous collision the background particles have no time to pass any energy to the colliding pair.) A neglect of $\Delta_E\to0$ leads to an inconsistency between the energy conservation (15) and the total energy (14). Indeed, the total energy (14) still includes $\mathrm{Re}\,T_R$ from which the mean energy gain arises. A consistent elastic approximation of collisions thus cannot be achieved by a simple neglect of the nonelasticity.

The link between the variational and the spectral concepts can be established if one is concerned with processes in which global conservation laws play the dominant role while individual trajectories of quasiparticles are of a marginal importance. In this hydrodynamic regime, it is possible to rearrange the mean energy gain into a mean-field-like contribution to the quasiparticle energy, ϵ_k^Δ. From a demand

$$\int\frac{dk}{(2\pi)^3}\epsilon_k^\Delta\frac{\partial\tilde{f}_k}{\partial t}=-\int dP\Delta_E, \tag{16}$$

and a variational form of the energy gain

$$\Delta_E=-\frac{1}{2}\frac{\partial\phi}{\partial t}=\int\frac{dk}{(2\pi)^3}\frac{\delta\phi}{\delta\tilde{f}_k}\frac{\partial\tilde{f}_k}{\partial t}, \tag{17}$$

one finds the mean-field-like correction as

$$\epsilon_k^{\Delta}=\frac{1}{2}\int dP\,\frac{\delta\phi}{\delta\tilde{f}_k}. \tag{18}$$

After a substitution of Eq. (16) into Eq. (15), a comparison with Eq. (7) shows that the mean-field-like correction is exactly the rearrangement energy, $\epsilon_k^{\Delta}=\epsilon_k^{\mathrm{re}}$. From Eq. (18) follows that the rearrangement energy describes the effect of the time-dependent Pauli blocking on the scattering phase shift ϕ.

Relation (16) shows an advantage of the variational concept. The elastic collision integral $\propto\delta(\tilde{\epsilon}_k+\tilde{\epsilon}_p-\tilde{\epsilon}_{k-q}-\tilde{\epsilon}_{p+q})$ with the variational quasiparticle energy $\tilde{\epsilon}=\epsilon+\epsilon^{\mathrm{re}}$, yields the same energy conservation as the nonelastic one, $\propto\delta(\epsilon_k+\epsilon_p-\epsilon_{k-q}-\epsilon_{p+q}-2\Delta_E)$, with the spectral quasiparticle energy. Without a sacrifice of the energy conservation, one can thus circumvent an inconvenience of nonelastic collision integrals by an incorporation of the rearrangement energy.

In summary, we would like to stress the answer to the question, Which of the quasiparticle concepts is more suitable for highly nonequilibrium systems? The spectral concept offers the more elaborate description of the system dynamics provided that the collision integral includes the collision delay and the energy gain. The instantaneous and elastic picture of collisions cannot capture such features of binary correlations like the correlated density. The mean energy gain, however, can be mimicked by the rearrangement energy included in the variational concept.

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